Notes 3.17 sec math 3H

Polar Equations and Graphs

Rectangular coordinate system – ordered pairs – (x, y)

Polar Coordinate system: select a point, called the pole, and then a ray with vertex at the pole, called the polar axis. A point P in a polar coordinate system is represented by an ordered pair of number (r, θ) If r > 0 then r is the distance of the point from the pole. θ is an angle (in degrees or radians) formed by the polar axis and a ray from the pole through the point.
(r, θ) is the polar coordinates of the point.
A point with polar coordinates (r, θ) also can be represented by either of the following
(r, θ+ 2*k*π) or (-r, θ + π+2*k*π) ***k* is any integer**
The polar coordinates of the pole are (0, θ) , where θ can be any angle.

Conversion from Polar Coordinates to Rectangular Coordinates
If P is a point with polar coordinates (r, θ), the rectangular coordinates (x, y) of P are given by
x = r cos θ y= r sin θ

Example:
P (3, 5π/6) Q (2, -200°)

Convert from Rectangular Coordinates to Polar Coordinates
Examples:
P (-1, 1) Q (-3, 0)
$r = \sqrt{x^2 + y^2}$
$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

Steps for converting from rectangular to Polar
Always plot the point (x, y) first
To find r, compute the distance from the origin to (x, y)
To find θ compute reference angle *a* of θ, $a = \tan^{-1}\left|\frac{y}{x}\right|$, *if* $x \neq 0$ then use your illustration to find θ.

Polar Equations and Graphs

An equation whose variables are polar coordinates is called a **polar equation.** The **graph of a polar equation** consists of all points whose polar coordinates satisfy the equation.

Identify and graph a polar equation.

Lines
Description: Line passing through the pole making an angle a with the polar axis.
Rectangular equation: y = (tan a)x x=a y=b
Polar Equation: θ = a r cosθ = a r sinθ = b
Typical graph

Circles
Description: Center at the pole, radius *a*
Rectangular equation: $a^2 = x^2 + y^2 \quad a > 0$
Polar Equation: $r = a \qquad a > 0$
Typical graph

Description: Passing through the pole tangent to the line $\theta=\pi/2$, center on the polar axis, radius *a*
Rectangular equation: $\pm 2ax = x^2 + y^2 \quad a > 0$
Polar Equation: $r = \pm 2a\cos\theta \qquad a > 0$
Typical graph

Circle: Description: Passing through the pole tangent to the polar axis, center on the line $\theta = \pi/2$, radius *a*
Rectangular equation: $\pm 2ay = x^2 + y^2 \quad a > 0$
Polar Equation: $r = \pm 2a\sin\theta \qquad a > 0$
Typical graph

Other Equations:
Rose Curves:
$r=a \sin (n\theta)$ or $r=a \cos(n\theta)$ n is an integer greater than 1
If n is odd there are n petals.
If n is even there are 2n petals.

Limacon Curves
$r = a \pm b\sin\theta \quad and \quad r = a \pm b\cos\theta$
Limacon with an inner loop: $0 < a < b$
Cartioid: $a = b \quad a > 0$
Limacon without an inner loop: $0 < b < a$

Lemniscate curve: $r^2 = a^2 \sin 2\theta \quad and \quad r^2 = a^2 \cos 2\theta \quad a > 0$
Spiral of Archimedes: $r = \theta$

Converting a polar equation to a rectangular equation.

$$r = 2\cos\theta$$

Converting a rectangular equation to a polar equation.

y = 3x -2

$$x^2 + y^2 = 9$$